Electronic feedback control of the intensity noise of a single-frequency intracavity-doubled laser

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We present a fully quantum model to process the intensity noise reduction of a single-frequency intracavity-doubled laser with an electronic feedback loop connected directly to the pump current of a laser diode. Adding an electronic feedback term to the quantum Langevin equations for a free-running single-frequency-doubling laser yields an analytical expression for the intensity noise spectrum of this system. Our previous experimental results with a Nd:YVO_4/KTP green laser are basically in agreement with the theoretical calculation.

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1. INTRODUCTION

Single-frequency light sources with low-intensity noise are useful for many applications, such as high-sensitivity measurements, high-precision interferometry, precision spectroscopy, and optical communications. Laser-diode-(LD-) pumped solid-state lasers and intracavity frequency lasers are well known as efficient sources for the generation of intensity-stable single-frequency radiation.[1–4] In practical LD-pumped single-frequency laser systems, however, the intensity noise spectrum has a resonance, that is, an underdamped driven second-order oscillator, known as resonant relaxation oscillation (RRO).[5] The low-frequency part of the intensity-noise spectrum below the RRO is determined by the laser's pump noise, whereas the RRO is driven by vacuum fluctuations, dipole fluctuations, and intracavity losses. A significant suppression of relaxation oscillations in diode-pumped single-frequency lasers has been achieved by various means, such as stabilizing laser intensity by means of electronic feedback loops,[6–9] injection locking of a laser to an intensity-stable master laser,[10,11] and a combination of both techniques.[12] The intensity noise at low frequencies can be reduced when the pump noise is suppressed.[13,14]

Recently technological progress has offered the possibility of achieving single-frequency operation of LD-pumped lasers with quite low intensity noise. Systems that produce several watts of harmonic light are already available from some vendors. Previously the authors and others[15] derived analytical expressions for the intensity noise spectra of a free-running single-frequency intracavity-doubled laser, using a linearized input–output method. Compared with the single-frequency laser, in the intensity noise spectrum of a single-frequency-doubling laser RRO is not present, but there is still a large amount of noise that results from an overdamped driven second-order oscillator during nonlinear conversion.[15] An experimental result of reducing the intensity noise of an intracavity frequency doubler by means of an optoelectronic feedback circuit directly connected to the pump source of the LD was presented recently by the authors and others.[16] In this paper we propose a quantum-theoretical model with which to process the system of electronic feedback to the pump source of an intracavity frequency doubler. Adding an electronic feedback term to the quantum Langevin equations for the free-running single-frequency-doubling laser produces the analytical expression for the intensity noise spectrum of this system. We compare the experimental results obtained with a Nd:YVO_4/KTP green laser with the theoretical calculation, and good agreement is shown.

This paper is arranged as follows: In Section 2 we introduce a linearized model of the intracavity frequency doubler that involves all quantum noise sources. A specific term with which to describe the feedback is introduced in Section 3. Theoretical analyses for a Nd:YVO_4/KTP green laser based on the proposed model are presented in Section 4, and a comparison of the calculation results with the experimental data is given in Section 5. A brief summary of the paper is found in Section 6.

2. LINEARIZED OPERATOR EQUATION OF MOTION

In a previous publication[15] a detailed derivation of the intensity-noise spectrum of a free-running single-frequency-doubling laser that was achieved with a full quantum model based on the Langevin approach was described. Here, we first briefly recall the method and then extend it to the laser with a feedback loop. We consider the active single-frequency and frequency-doubled ring laser shown in Fig. 1. All cavity mirrors have high reflectivity for the fundamental frequency ω_0. The laser

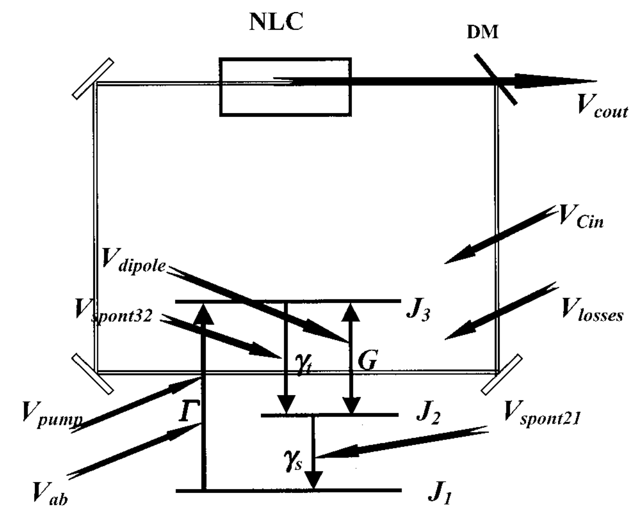

Fig. 1. Schematic of a quantum model description of the diode-pumped single-frequency doubling laser. γ_t and γ_s are spontaneous-emission rates; Γ is the pump rate; G is the stimulated-emission coefficient; $V_{c_{\mathrm{out}}}$ is the output harmonic noise; $V_{c_{\mathrm{in}}}$ is second-harmonic input noise; V_{pump} is the noise entering the laser from its pump source; V_{ab} is quantum (or vacuum) noise that is due to imperfect absorption in the pump field; V_{spont32} and V_{spont21} are noises from spontaneous emission; V_{dipole} is the noise from dipole fluctuations; and V_{losses} is the noise from intracavity losses. DM is a dichroic mirror (reflectivity, $R \sim 1$ for the fundamental and $R \sim 0$ for the harmonic); NLC, nonlinear crystal.

medium and an optical nonlinear crystal were placed inside the laser cavity. The harmonic light generated from the nonlinear crystal passes through one of the cavity mirrors, which was antireflection coated for harmonic frequency $2\omega_0$. To ensure single-mode oscillation, we inserted a Faraday rotator and a polarization-selecting device into the cavity. We obtained the linearized fluctuation equations by expanding the operators about their semiclassical values. Resorting to adiabatical elimination of the equation for the upper pump level, we further simplified the set of equations for the four-level atoms, which led to unidirectional pumping of the electrons from the ground state directly to the upper lasing level. Finally, the set of equations could be used to describe a three-level laser with noise inputs. The electrons were pumped from the ground state to the upper lasing level at a rate Γ by pump field $\hat{B}$. Fluctuation δX_B in the amplitude of the pump field introduces a source of noise during pumping. As not all the pump field was absorbed, an extra vacuum noise, δX_q, must have been involved. The lasing transition between levels 2 and 3 relates to two sources of quantum noise: the first one, δX_{C_t}, is due to spontaneous emission between the lasing levels, which occurs at a rate γ_t; the second, δX_{C_p}, is due to collision- or lattice-induced dephasing of the lasing levels at a rate γ_p. Electrons decaying out of the lower lasing level at a rate γ_s also give rise to spontaneous emission noise, δX_{C_s}. Other noise sources included in the model are the input vacuum fluctuation δX_c of the second-harmonic mode and δX_{A_l}, which is derived from total losses κ_l of the fundamental lasing mode within the cavity. $\hat{a}$ is the fundamental lasing mode in the cavity, and the $\hat{c}_{\mathrm{out}}$ is the second-harmonic output.

The semiclassical equations of motion for the atomic populations, intracavity fundamental mode, and second-harmonic mode amplitude are

$$\begin{aligned}
\dot{\alpha}_0 &= \frac{G}{2}(J_3 - J_2)\alpha_0 - \kappa_l \alpha_0 - \tilde{\mu}\alpha_0{}^*\alpha_0{}^2,\\
\dot{J}_1 &= -\Gamma J_1 + \gamma_s J_2,\\
\dot{J}_2 &= G(J_3 - J_2)\alpha_0\alpha_0{}^* + \gamma_t J_3 - \gamma_s J_2,\\
\dot{J}_3 &= -G(J_3 - J_2)\alpha_0\alpha_0{}^* - \gamma_t J_3 + \Gamma J_1,\\
J_1 + J_2 + J_3 &= 1,\\
c_{\mathrm{out}} &= \sqrt{\tilde{\mu}}\alpha_0{}^2,
\end{aligned} \tag{1}$$

where J_i is the population of level i scaled by the number of atoms N, and α_0 is the amplitude of the lasing fundamental mode per root N. $\tilde{\mu} = N\mu$, where μ is the dipole coupling strength between the lasing mode and the second-harmonic mode. Rate G is the stimulated-emission rate per photon in the lasing mode, which is proportional to the stimulated-emission cross section ε_s:

$$G = \varepsilon_s \rho c, \tag{2}$$

where ρ is the atomic density and c is the speed of light in the medium. Setting the time derivatives to zero in Eq. (1), we obtained the semiclassical steady-state solution and found the stable steady-state operating point of the laser.

The linearized operator equations for the fluctuations of the laser parameters near their semiclassical values are[15]

$$\begin{aligned}
\delta\dot{\hat{X}}_a &= G(\delta\hat{\sigma}_3 - \delta\hat{\sigma}_2)\alpha_0 - \tilde{\mu}\alpha_0{}^*\alpha_0\delta\hat{X}_a - \tilde{\mu}\alpha_0{}^2\delta\hat{X}_a\\
&\quad + \sqrt{2\kappa_l}\delta\hat{X}_{A_l} + 2\sqrt{\tilde{\mu}}\alpha_0{}^*\delta\hat{X}_c\\
&\quad - [G(J_3 + J_2)]^{1/2}\delta\hat{X}_{C_p},\\
\delta\dot{\hat{\sigma}}_1 &= -\Gamma\sqrt{1-\eta}\delta\hat{\sigma}_1 + \gamma_s\delta\hat{\sigma}_2 - \sqrt{\Gamma J_1\eta}\delta\hat{X}_B\\
&\quad + \sqrt{\gamma_s J_2}\delta\hat{X}_{C_s} + [\Gamma(1-\eta)J_1]^{1/2}\delta\hat{X}_q,\\
\delta\dot{\hat{\sigma}}_2 &= G(\delta\hat{\sigma}_3 - \delta\hat{\sigma}_2)\alpha_0{}^2 + G(J_3 - J_2)\alpha_0\delta\hat{X}_a\\
&\quad + \gamma_t\delta\hat{\sigma}_3 - \gamma_s\delta\hat{\sigma}_2 + \sqrt{\gamma_s J_2}\delta\hat{X}_{C_s} - \sqrt{\gamma_t J_3}\delta\hat{X}_{C_t}\\
&\quad - [G(J_3 + J_2)]^{1/2}\alpha_0\delta\hat{X}_{C_p},\\
\delta\dot{\hat{\sigma}}_3 &= -G(\delta\hat{\sigma}_3 - \delta\hat{\sigma}_2)\alpha_0{}^2 - G(J_3 - J_2)\alpha_0\delta\hat{X}_a\\
&\quad - \gamma_t\delta\hat{\sigma}_3 + \Gamma\sqrt{1-\eta}\delta\hat{\sigma}_1 + \sqrt{\Gamma J_1\eta}\delta\hat{X}_B\\
&\quad + \sqrt{\gamma_s J_2}\delta\hat{X}_{C_s} - [\Gamma(1-\eta)J_1]^{1/2}\delta\hat{X}_q\\
&\quad + \sqrt{\gamma_t J_3}\delta\hat{X}_{C_t} + [G(J_3 + J_2)]^{1/2}\alpha_0\delta\hat{X}_{C_p},
\end{aligned} \tag{3}$$

where $\delta\hat{X}_a$ is the operator for the fluctuations of the fundamental lasing mode amplitude quadrature, $\delta\hat{\sigma}_i$ is the operator for the fluctuations of the atomic population about J_i, and η is the absorption efficiency of pump field. According to the boundary condition of the second-harmonic output field, the second-harmonic amplitude quadrature fluctuation is given by

$$\delta\hat{X}_{c_{\mathrm{out}}} = 2\sqrt{\tilde{\mu}}\alpha_0\delta\hat{X}_a - \delta\hat{X}_c. \tag{4}$$

Hence the expression for the quadrature amplitude fluctuations of second-harmonic output of the free-running laser is obtained in terms of the input field fluctuations in frequency space:

$$\delta X_{\mathrm{free\_}c_{\mathrm{out}}} = W_0(\omega)\delta X_B + W_1(\omega)\delta X_c + W_2(\omega)\delta X_q + W_3(\omega)\delta X_{C_s} + W_4(\omega)\delta X_{C_t} + W_5(\omega)\delta X_p + W_6(\omega)\delta X_{A_l}, \tag{5}$$

where the absence of a circumflex indicates a Fourier transform and

$$W_0(\omega) = \frac{2\sqrt{\tilde{\mu}}\alpha_0\sqrt{\eta\Gamma J_1}G\alpha_0(i\omega + \gamma_s - \gamma_t)}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})},$$

$$W_1(\omega) = \frac{4\tilde{\mu}\alpha_0{}^2\zeta(\omega)}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})} - 1,$$

$$W_2(\omega) = \left(\frac{1-\eta}{\eta}\right)^{1/2} W_0(\omega),$$

$$W_3(\omega) = \frac{2\sqrt{\tilde{\mu}}\alpha_0 G\alpha_0\sqrt{\gamma_s J_2}(i\omega + 2\tilde{\Gamma} + \gamma_t)}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})},$$

$$W_4(\omega) = \frac{2\sqrt{\tilde{\mu}}\alpha_0\sqrt{\gamma_t J_3}G\alpha_0(i\omega + 2\tilde{\Gamma} + \gamma_s)}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})},$$

$$W_5(\omega) = \frac{2\sqrt{\tilde{\mu}}\alpha_0[G(J_3 + J_2)]^{1/2}[G\alpha_0{}^2(i\omega + 2\tilde{\Gamma} + \gamma_s) - \zeta(\omega)]}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})},$$

$$W_6(\omega) = \frac{2\sqrt{\tilde{\mu}}\alpha_0\sqrt{2k_l}\zeta(\omega)}{(i\omega + 2\tilde{\mu}\alpha_0{}^2)\zeta(\omega) + G^2\alpha_0{}^2(J_3 - J_2)(2i\omega + \gamma_s + 2\tilde{\Gamma})},$$

$$\zeta(\omega) = (i\omega + \tilde{\Gamma})(i\omega + \gamma_s + 2G\alpha_0{}^2 + \gamma_t) + \gamma_s(G\alpha_0{}^2 + \gamma_t), \tag{6}$$

where $\tilde{\Gamma} = \Gamma\sqrt{1-\eta}$. The second-harmonic amplitude noise spectrum ($V_{\mathrm{free\_}c_{\mathrm{out}}}$) is

$$\begin{aligned} V_{\mathrm{free\_}c_{\mathrm{out}}}(\omega) &= \langle|\delta X_{\mathrm{free\_}c_{\mathrm{out}}}|^2\rangle \\ &= |W_0(\omega)|^2 V_{\mathrm{pump}} + |W_1(\omega)|^2 V_{c_{\mathrm{in}}} \\ &\quad + |W_2(\omega)|^2 V_{\mathrm{ab}} \\ &\quad + |W_3(\omega)|^2 V_{\mathrm{spont21}} |W_4(\omega)|^2 V_{\mathrm{spont32}} \\ &\quad + |W_5(\omega)|^2 V_{\mathrm{dipole}} + |W_6(\omega)|^2 V_{\mathrm{losses}}. \end{aligned} \tag{7}$$

A variety of noise sources in Eq. (7) is separately expressed, term by term: $V_{c_{\mathrm{in}}} = \langle|\delta X_c|^2\rangle$ is the second-harmonic input noise; $V_{\mathrm{ab}} = \langle|\delta X_q|^2\rangle$ is quantum (or vacuum) noise owing to imperfect absorption to pump field; $V_{\mathrm{pump}} = \langle|\delta X_B|^2\rangle$ is the intensity noise of the pump source; $V_{\mathrm{spont21}} = \langle|\delta X_{C_s}|^2\rangle$ and $V_{\mathrm{spont32}} = \langle|\delta X_{C_t}|^2\rangle$ are the spontaneous-emission noise from level $|2\rangle$ to level $|1\rangle$ and from level $|3\rangle$ to level $|2\rangle$, respectively; $V_{\mathrm{dipole}} = \langle|\delta X_{C_p}|^2\rangle$ is the dipole fluctuation noise between level $|3\rangle$ and level $|2\rangle$; and $V_{\mathrm{losses}} = \langle|\delta X_{A_l}|^2\rangle$ is the noise introduced from the fundamental mode intracavity losses. The second-harmonic amplitude noise spectrum $V_{\mathrm{free\_}c_{\mathrm{out}}}$ is normalized to the quantum-noise limit. All input quantum-noise sources are on the level of the quantum-noise limit, $V_{c_{\mathrm{in}}} = V_{\mathrm{ab}} = V_{\mathrm{spont21}} = V_{\mathrm{spont32}} = V_{\mathrm{losses}} = 1$. However, pump noise spectrum V_{pump} depends on the noise of the pump source.

3. SECOND-HARMONIC AMPLITUDE NOISE SPECTRUM INCLUDING OPTOELECTRONIC FEEDBACK

In the case including optoelectronic feedback connected to the pump current of the LD, pump field $\hat{P}$ can be written with two terms:

$$\hat{P} = \hat{B} + \delta\hat{E}, \tag{8}$$

where $\hat{B}$ is pump field without feedback and $\delta\hat{E}$ is the additional field introduced by the feedback. Field $\delta\hat{E}$ does not change the dc operating point of the laser, which is governed solely by $\hat{B}$. The amplitude fluctuations of the pump field can be found by linearizing $\hat{B}$ about its semiclassical average value $\bar{B}$:

$$\begin{aligned} \hat{P} &= \bar{B} + \delta\hat{B} + \delta\hat{E} \\ \Rightarrow \delta\hat{P} &= \delta\hat{B} + \delta\hat{E} \\ \Rightarrow \delta\hat{X}_P &= \delta\hat{X}_B + \delta\hat{X}_E. \end{aligned} \tag{9}$$

Field $\delta\hat{E}$ may be expressed as a convolution of the time response of the feedback electronics, $k(t)$, and the ac component of the in-loop photocurrent, $\delta\hat{i}_1(t)$. We assume that there is an optical loss β in the diode laser, which attenuates the feedback signal and introduces additional vacuum noise $\delta\hat{v}_\beta$, so $\delta\hat{E}$ is written as follows:

$$\delta\hat{E} = -\sqrt{\beta}\int_{-\infty}^{\infty} k(v)\delta\hat{i}_1(t-v)dv - \sqrt{1-\beta}\delta\hat{v}_\beta, \tag{10}$$

where the minus is involved with negative feedback on the pump source. Photocurrent $\hat{i}_1$ may be expressed in terms of second-harmonic output $\hat{c}_{\text{out}}$, beam-splitter ratio ϵ, the detector efficiency η_D:

$$\begin{aligned}\hat{i}_1 = &\{\sqrt{\epsilon\eta_D}\hat{c}_{\text{out}} + [\eta_D(1-\epsilon)]^{1/2}\delta\hat{v}_s + \sqrt{1-\eta_D}\delta\hat{v}_D\}\\ &\times\{\sqrt{\epsilon\eta_D}\hat{c}_{\text{out}} + [\eta_D(1-\epsilon)]^{1/2}\delta\hat{v}_s + \sqrt{1-\eta_D}\delta\hat{v}_D\},\end{aligned} \tag{11}$$

where vacuum fluctuations $\delta\hat{v}_s$ and $\delta\hat{v}_D$ result from the beam splitter and the in-loop photodetector, respectively. Field $\hat{c}_{\text{out}}$ is expanded about the semiclassical value $\bar{c}_{\text{out}}$, i.e., $\hat{c}_{\text{out}} = \bar{c}_{\text{out}} + \delta\hat{c}_{\text{out}}$. Retaining only first-order fluctuation terms yields the following fluctuations of the in-loop photocurrent:

$$\begin{aligned}\delta\hat{i}_1 = &\sqrt{\epsilon\eta_D}\sqrt{\bar{\mu}}\alpha_0^2\{\sqrt{\epsilon\eta_D}\delta\hat{X}_{c_{\text{out}}} + [\eta_D(1-\epsilon)]^{1/2}\delta\hat{X}_{v_s}\\ &+ \sqrt{1-\eta_D}\delta\hat{X}_{v_D}\},\end{aligned} \tag{12}$$

where we have taken $\bar{c}_{\text{out}} = \sqrt{\bar{\mu}}\alpha_0^2$. Amplitude quadrature fluctuation $\delta\hat{X}_{i_1}$ and $\delta\hat{X}_E$ are then given by

$$\begin{aligned}\delta\hat{X}_{i_1} = &\sqrt{\epsilon\eta_D}\delta\hat{X}_{c_{\text{out}}} + [\eta_D(1-\epsilon)]^{1/2}\delta\hat{X}_{v_s}\\ &+ \sqrt{1-\eta_D}\delta\hat{X}_{v_D},\end{aligned} \tag{13}$$

$$\begin{aligned}\delta\hat{X}_E = &-\sqrt{\epsilon\eta_D}\sqrt{\bar{\mu}}a_0^2\int_{-\infty}^{+\infty} k(v)\delta\hat{X}_{i_1}(t-v)dv\\ &- \sqrt{1-\eta_D}\delta\hat{X}_{v_\beta},\end{aligned} \tag{14}$$

respectively. Substituting Eqs. (14), (13), and (9) into Eqs. (3), and using boundary condition Eq. (4), we solved these equations in Fourier space to yield the amplitude quadrature fluctuation of the second-harmonic output:

$$\begin{aligned}\delta X_{fd\_c_{\text{out}}} = &\left\{W_0(\omega)\left[\delta X_B - \left(\frac{1-\beta}{\eta}\right)^{1/2}\delta X_\beta\right] - W_0(\omega)H(\omega)\right.\\ &\times\left[\left(\frac{1-\epsilon}{\epsilon}\right)^{1/2}\delta X_{v_s} + \left(\frac{1-\eta_D}{\epsilon\eta_D}\right)^{1/2}\delta X_{v_D}\right]\\ &+ W_1(\omega)\delta X_c + W_2(\omega)\delta X_q + W_3(\omega)\delta X_{C_s}\\ &+ W_4(\omega)\delta X_{C_t} + W_5(\omega)\delta X_{C_p}\\ &\left.+ W_6(\omega)\delta X_{A_l}\right\}\Big/[1 + W_0(\omega)H(\omega)],\end{aligned} \tag{15}$$

where the form of function $W_n(\omega)$ was given in Section 2 and $H(\omega)$ stands for

$$H(\omega) = 2\sqrt{\beta}\epsilon\eta_D\sqrt{\bar{\mu}}\alpha_0^2K(\omega). \tag{16}$$

Function $H(\omega)$ could then be identified as the transfer function of the feedback system to summarize the total effect of the beam splitter, the control electronics, and the LD efficiency. Function $W_0(\omega)$ is the transfer function of the free-running laser with respect to pump noise. As was mentioned above for Eqs. (6), function $W_i(\omega)$ ($i = [1,\ldots,6]$) is the free-running laser transfer function for the other various quantum-noise terms.

The noise spectra of the in- and out-of-loop fields that emerges from the beam splitter, $\hat{c}_1$ and $\hat{c}_2$, are expressed in terms of $\hat{X}_{fd\_c_{\text{out}}}$:

$$\begin{aligned}\delta X_{c_1} &= \sqrt{\epsilon}\delta X_{fd\_c_{\text{out}}} + \sqrt{1-\epsilon}\delta X_{v_s},\\ \delta X_{c_2} &= \sqrt{1-\epsilon}\delta X_{fd\_c_{\text{out}}} - \sqrt{\epsilon}\delta X_{v_s}.\end{aligned} \tag{17}$$

Combining Eqs. (15) and (17), we found the spectra of the in- and out-of-loop fields:

$$V_{c_1} = \frac{\epsilon[V_{\text{free\_}c_{\text{out}}} + |W_0(\omega)|^2(1-\beta/\eta)] + |W_0(\omega)H(\omega)|^2(1-\eta_D/\epsilon\eta_D) + (1-\epsilon)V_{v_s}}{|1 + W_0(\omega)H(\omega)|^2}, \tag{18}$$

$$V_{c_2} = \frac{(1-\epsilon)[V_{\text{free\_}c_{\text{out}}} + |W_0(\omega)|^2(1-\beta/\eta) + |W_0(\omega)H(\omega)|^2)(1-\eta_D/\epsilon\eta_D)] + (V_{v_s}/\epsilon)|\epsilon + W_0(\omega)H(\omega)|}{|1 + W_0(\omega)H(\omega)|^2}, \tag{19}$$

where $V_{\text{free\_}c_{\text{out}}}$ is the noise spectrum of the free-running laser and V_{v_s} is the noise that is due to the injected vacuum at the beam splitter.

The out-of-loop field is our main interest, because this is the only observable output of the control loop. In the limit without feedback [ϵ, $H(\omega) \to 0$] and $\beta = 1$, the spectrum V_{c_2} is recovered to the free-running second-harmonic amplitude noise spectrum [Eq. (7)]. In the limit of large negative feedback, the out-of-loop noise was found to be

$$\lim_{H(\omega)\to\infty}(V_{c_2}) = \frac{(1-\epsilon)(1-\eta_D)}{\epsilon\eta_D} + \frac{V_{v_s}}{\epsilon}. \tag{20}$$

For a Poissonian vacuum at the beam splitter ($V_{v_s} = 1$) the high gain limit of the out-of-loop noise is always greater than 1; i.e., it is always super-Poissonian. This result is in agreement with the conclusions of other quantum-feedback models.[17–19] The cause of this behavior lies in the vacuum fluctuations introduced by the beam splitter. Because the noise that is due to the beam splitter in the out-of-loop field is out of phase with the noise introduced to the in-loop field [see Eqs. (17)], the negative feedback therefore amplifies the beam-splitter vacuum noise in the out-of-loop field, which precludes reaching sub-Poissonian intensity statistics. The high-gain limit of the in-loop field is

$$\lim_{H(\omega)\to\infty} (V_{c_1}) = \frac{1 - \eta_D}{\eta_D}. \tag{21}$$

The noise of the in-loop field can therefore become sub-Poissonian, depending on the value of η_D. This result is also in agreement with other quantum-feedback models,[17–19] but this field is not a free field and does not obey the commutators of a free field.

4. THEORETICAL ANALYSES OF THE NOISE REDUCTION FOR A Nd:YVO$_4$/KTP GREEN LASER

The Nd:YVO$_4$/KTP green laser always runs in a regime where the rate of decay out of lower lasing level γ_s is much larger than stimulated-emission rate $G\alpha_0{}^2$, pump rate Γ, and rate γ_t of spontaneous emission between lasing levels. The corresponding parameter values for the Nd:YVO$_4$/KTP green laser are listed in Table 1. In this case the pump noise transfer function is simplified to

$$W_0(\omega) = \frac{2(\tilde{\mu}\alpha_0{}^2\Gamma\,\eta J_1)^{1/2} G\alpha_0}{i\omega\gamma_L + (\omega_r{}^2 - \omega^2)}, \tag{22}$$

where

$$\omega_r = [2\tilde{\mu}\alpha_0{}^2(G\alpha_0{}^2 + \tilde{\Gamma} + \gamma_t) + 2G\alpha_0{}^2(\tilde{\mu}\alpha_0{}^2 + \kappa_l)]^{1/2}, \tag{23}$$

Table 1. Properties of the Nd:YVO$_4$/KTP Single-Frequency Doubling Ring Laser

Property	Symbol	Value
Length of cavity	L	350 mm
Single-pass losses	δ_{cav}	2%
Maximum pump power	$P_{\max}$	2 W
Decay rate of the intracavity losses	k_1	$8.5 \times 10^6\ \text{s}^{-1}$
Number of lasing atoms	N	10^{17}
Decay rate of the upper lasing level	γ_t	$3 \times 10^3\ \text{s}^{-1}$
Decay rate of the lower lasing level	γ_s	$3.3 \times 10^7\ \text{s}^{-1}$
Simulated-emission rate per photon	G	$2 \times 10^{12}\ \text{s}^{-1}$
Maximum pump rate	Γ	$8\ \text{s}^{-1}$
Nonlinear conversion rate	$\tilde{\mu}$	$6 \times 10^{14}\ \text{s}^{-1}$

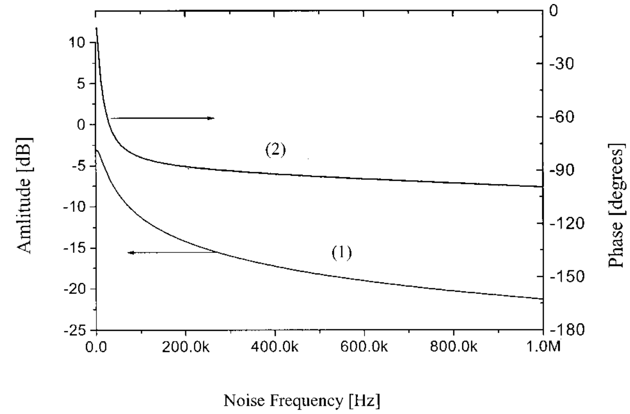

Fig. 2. Amplitude (1) and phase (2) of the pump noise transfer function for the single-frequency intracavity frequency-doubling laser.

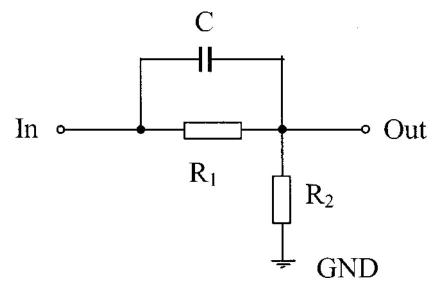

Fig. 3. Circuit of a phase advance filter. GND, the ground of the circuit.

$$\gamma_L = 2\tilde{\mu}\alpha_0{}^2 + G\alpha_0{}^2 + \tilde{\Gamma} + \gamma_t. \tag{24}$$

ω_r indicates the resonant frequency in the spectrum, and γ_L is the damping rate of the oscillations. Under these conditions, $W_0(\omega)$ is seen to be a second-order transfer function, which is similar to the expression for a damped, driven pendulum. Frequency ω_r and damping rate γ_L of the single-frequency-doubling laser include the term of nonlinear conversion $\tilde{\mu}\alpha_0{}^2$, so the frequency-doubling process will influence the behavior of a laser significantly. Usually damping rate γ_L is larger than ω_r; then the pump transfer function is the overdamped driven second-order oscillator and does not exhibit the phenomenon of RRO.[15,16] Plots of the phase and the amplitude of the pump transfer function for a single-frequency-doubling laser are shown in Fig. 2. There is no peak of RRO in the amplitude plot of the pump transfer function [curve (1)], and the phase changes smoothly, with no areas of abrupt change [curve (2)]. In our design to the feedback loop a second-order oscillator is considered. The requirement for designing a good feedback circuit should be to ensure that the magnitude of the open-loop gain is less than 1 when the phase of the open-loop gain reaches $-180°$; otherwise the feedback loop will be unstable, because of an enhancement of the spectral noise. Note also that a stable feedback loop may amplify noise if the open-loop gain approaches -1.

We introduce a phase advance filter as shown Fig. 3 to enhance the performance of the control loop. The form of the phase advance filter is

$$H_{ad}(\omega) = \mathrm{F}\frac{R_2(1 + i\omega CR_1)}{R_2(1 + i\omega CR_1) + R_1}, \tag{25}$$

where R_1 and R_2 are the resistances, C is the capacitor, and F is the dc gain of the phase advance filter. It has a maximum phase advance of ϕ_m at frequency ω_m, where

$$\tan \phi_m = \frac{R_1}{2[R_2(R_2 + R_1)]^{1/2}}, \tag{26}$$

$$\omega_m = [(R_2 + R_1)/C^2 {R_1}^2 R_2]^{1/2}. \tag{27}$$

A high-pass filter serves for the amplitude response of $H_{ad}(\omega)$. To optimize the feedback loop we chose frequency ω_m on a specific operating point where the open-loop transfer function $W_0(\omega)H_{ad}(\omega)$ has a magnitude of 1 by solving

$$|W_0(\omega_m)H_{ad}(\omega_m| = 1,$$

$$\omega_m = [(R_2 + R_1)/C^2 {R_1}^2 R_2]^{1/2}. \tag{28}$$

for capacitor C. We can freely choose the amount of phase advance ϕ_m by changing the resistance R_2 and R_1. Figure 4 shows the predicted effect of feedback for a typical Nd:YVO_4/KTP single-frequency-doubling laser with the theoretical analyses given above. Curve a is the noise spectrum of the free-running laser, i.e., $H_{ad}(\omega) = 0$. Curve b is a spectrum with feedback but without a phase advance filter, where we take $H_{ad}(\omega) = \mathsf{F}$; F is a real positive number. The spectrum with the optimized phase advance filter is shown in curve c. It is obvious that a considerable improvement in laser noise has been achieved.

5. COMPARISON OF THEORETICAL CALCULATION AND EXPERIMENTAL RESULTS FOR A Nd:YVO_4/KTP SINGLE-FREQUENCY-DOUBLING LASER

The detailed experimental arrangement and results for the noise spectrum measurement can be found in a previous publication.[16] The experimental arrangement for noise control and monitoring of a laser is shown in Fig. 5. We monitor in-loop and out-of-loop noise of the laser with photodetectors D_1 (in loop) and D_2 (out of loop) (FND-100 silicon photodiodes from EG&G). D_1 (Analog Modules 714A) has a large gain and a broad bandwidth, from 10 kHz to ~100 MHz. A transimpedance operational amplifier circuit in D_2 is used to convert photocurrent to voltage. We inject the error current directly into the DL from the driving circuit to minimize time delay. This circuit consists of a buffer (BUF634) followed by a 100-Ω resistor in parallel with a 1-nF capacitor, followed by a 4-μF capacitor, which ac couples the injected signal to prevent any change in the output power of the diode laser. A noise reduction circuit is employed to reduce the noise of the laser. The noise-reduction circuit used in the feedback loop consists of three noninverting amplifiers and a series of active filters to provide phase advance and gain. The phase advance filter can enhance the performance of the control loop, which includes second-order resonance. For the single-frequency Nd:YVO_4/KTP green laser the maximum phase advance is 34° at 300 kHz, and its feedback loop is more stable than the fundamental laser because the single-frequency intracavity frequency-doubling laser includes an overdamped driven second-order oscillator. Open-loop gain G starting from point A returning to point A was found to attain a maximum value of 8 dB at 100 kHz, thus providing an intensity noise reduction factor $1/|1 + G|$ of ~7 dB, and to have two unity gain points at ~200 Hz and ~200 kHz, with ~20° and ~100° phase margins. The noise reduction spectra are shown in Fig. 6. The noise at low frequency is reduced by 7 dB relative to that of the free-running laser. The noise is amplified near 300 kHz because that is where the open-loop gain

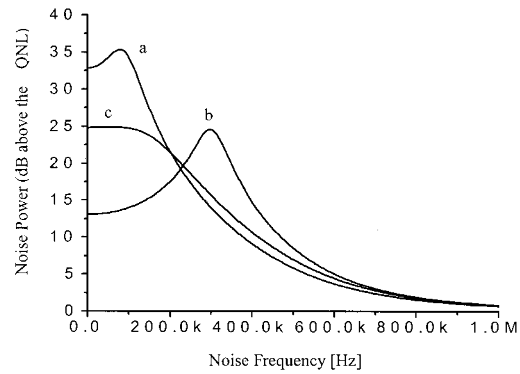

Fig. 4. Effect of feedback on intensity noise in a typical Nd:YVO_4/KTP single-frequency-doubling laser. a, Free-running laser spectrum; b, spectrum with feedback and without a phase advance filter; c, spectrum with feedback and with an optimized phase advance filter. Parameters used in the calculation were $V_{pump} = 10^6$, $\epsilon = 0.05$, and $\eta_1 = 0.9$. The phase advance is 45°.

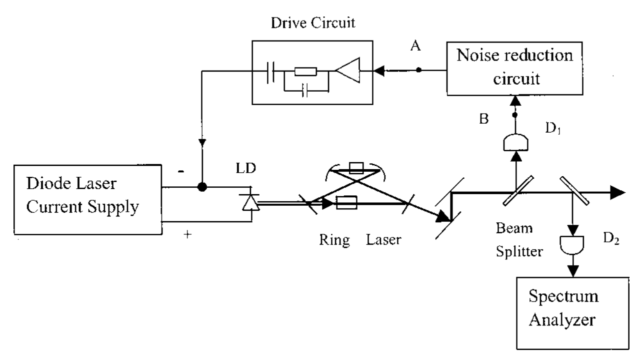

Fig. 5. Experimental arrangement for noise control and monitoring of the laser. A and B are the test points.

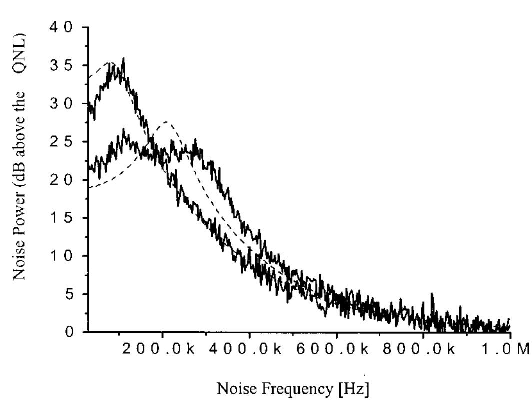

Fig. 6. Comparison of theoretical calculation and experimental result. QNL, quantum-noise limit.

approaches -1. To fit the experimental noise spectra to the numerical calculation we take

$$H(\omega) = \rho K(\omega)\exp(-i\omega\tau), \tag{29}$$

where $K(\omega)$ is the transfer function of the electronics in the feedback loop. Parameter ρ is used to express the change in gain that is due to the detected quantities of light and the efficiency of the LDs. Parameter τ is a time delay introduced by the LDs and the optical path that is not included in $K(\omega)$. Time delays are important, as they introduce a phase lag that counteracts the required phase advance. In the numerical calculation we use Eq. (25) as the transfer function $K(\omega)$, in which resistance R_1 and R_2 and capacitor C are all experimental parameters but ρ and $\omega\tau$ are regarded as constant parameters for different frequencies to reach the optimum fitness with the experimental noise spectra. Figure 6 shows the intensity noise spectra of the harmonic field measured experimentally and the result of numerical calculation with the formula in Section 4. We can see that they are qualitatively in agreement. However, in fact ρ and $\omega\tau$ are not exact constants and cannot easily be experimentally measured, because parameter ρ depends on the detected quantities of light and the efficiency of the LDs and time delay τ depends on the LD driver, the LD, and the optical path. The shape of the control loop of the phase advance filter used in the experiment is not exactly in agreement with the theoretical design values, and phase delay $\omega\tau$ has a frequency dependence. In the calculation we merely regard them as constants, that is, as the origins of this discrepancy between theoretical and experimental values, especially at low frequency and near 300 kHz.

6. CONCLUSIONS

For the first time of which we are aware, the behavior of a laser-diode-pumped single-frequency $Nd:YVO_4/KTP$ green laser with electronic feedback to the pump source was investigated theoretically. We obtained an analytical expression for the intensity noise spectrum by combining a quantum Langevin approach to a single-frequency-doubling laser with a model of the feedback loop. The good agreement between the theoretical prediction and the experimental results for a $Nd:YVO_4/KTP$ green laser demonstrated the utility of the model presented here.

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